

# 线性相关性

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例：设

$$\begin{aligned} \alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 \\ -2 \\ 0 \\ 1 \\ -1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \alpha_4 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \alpha_5 = \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -2 \end{pmatrix}, \\ \alpha_6 = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ -1 \\ -1 \end{pmatrix}, \alpha_7 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \alpha_8 = \begin{pmatrix} -1 \\ -1 \\ 0 \\ -1 \\ -1 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \alpha_9 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \alpha_{10} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \end{aligned}$$

判别  $\alpha_1, \alpha_2, \dots, \alpha_{10}$  是否线性相关？如果线性相关，求一个极大线性无关组，并写出其他向量由该极大无关组表出的表达式。

解：令  $A = (\alpha_1, \alpha_2, \dots, \alpha_{10})$  在命令窗口输入：

`B=rref(A)`

得到

`B =`

1	0	0	0	0	0	0	0	215/426	11/71
0	1	0	0	0	0	0	0	93/172	48/71
0	0	1	0	0	0	0	0	1/213	11/71
0	0	0	1	0	0	0	0	-131/213	-21/71
0	0	0	0	1	0	0	0	-85/213	-12/71
0	0	0	0	0	1	0	0	-55/184	-14/71
0	0	0	0	0	0	1	0	79/142	-10/71
0	0	0	0	0	0	0	1	62/689	-14/71

由此可见,  $\alpha_1, \alpha_2, \dots, \alpha_8$  是一个极大无关组, 且

$$\alpha_9 = 215/426\alpha_1 + 93/172\alpha_2 + 1/213\alpha_3 - 131/213\alpha_4 - 85/213\alpha_5 - 55/184\alpha_6 + 79/142\alpha_7 + 62/689\alpha_8$$

$$\alpha_{10} = 11/71\alpha_1 + 48/71\alpha_2 + 11/71\alpha_3 - 21/71\alpha_4 - 12/71\alpha_5 - 14/71\alpha_6 - 10/71\alpha_7 - 14/71\alpha_8$$

习题: 设

$$\alpha_1 = \begin{pmatrix} 10 \\ 2 \\ 6 \\ 5 \\ 9 \\ 8 \\ 5 \\ 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 8 \\ 4 \\ 6 \\ 8 \\ 9 \\ 7 \\ 2 \\ 4 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 9 \\ 9 \\ 4 \\ 9 \\ 1 \\ 4 \\ 8 \\ 0 \end{pmatrix}, \alpha_4 = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 6 \\ 3 \\ 2 \\ 0 \\ 7 \end{pmatrix}, \alpha_5 = \begin{pmatrix} 4 \\ 9 \\ 5 \\ 4 \\ 8 \\ 5 \\ 2 \\ 7 \end{pmatrix},$$

$$\alpha_6 = \begin{pmatrix} 8 \\ 0 \\ 7 \\ 4 \\ 8 \\ 5 \\ 7 \\ 4 \end{pmatrix}, \alpha_7 = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 7 \\ 3 \\ 5 \\ 2 \\ 7 \end{pmatrix}, \alpha_8 = \begin{pmatrix} 4 \\ 9 \\ 9 \\ 6 \\ 5 \\ 9 \\ 8 \\ 6 \end{pmatrix}, \alpha_9 = \begin{pmatrix} 8 \\ 7 \\ 3 \\ 3 \\ 3 \\ 5 \\ 7 \\ 3 \end{pmatrix}, \alpha_{10} = \begin{pmatrix} 8 \\ 6 \\ 4 \\ 7 \\ 5 \\ 4 \\ 7 \\ 6 \end{pmatrix},$$

判别  $\alpha_1, \alpha_2, \dots, \alpha_{10}$  是否线性相关? 如果线性相关, 求一个极大线性无关组, 并写出其他向量由该极大无关组表出的表达式.